On the Holomorphic Structure of a Low Energy Supersymmetric Wilson Effective Action

T.E. Clark and S.T. Love
Department of Physics
Purdue University
West Lafayette, IN 47907-1396

Abstract

The Wilson (exact) renormalization group equations are used to determine the evolution of a general low energy N=1 supersymmetric action containing a U(1) gauge vector multiplet and a neutral chiral multiplet. The effective theory evolves towards satisfying a fixed relation where the Kähler potential and effective gauge coupling are obtained from a N=2 supersymmetric holomorphic prepotential.

As a consequence of the non-renormalization properties of their radiative corrections, the low energy structure of many supersymmetric models can be exactly determined. This attribute was first displayed in calculations of the superpotential in purely perturbative models [1][2]. Subsequently, it was recognized that by combining various symmetries with the holomorphic dependence on the fields and parameters of a model, the superpotential can also be completely (non-perturbatively) obtained even in the framework of strongly interacting gauge theories [3]. For N=2 supersymmetric theories, the low energy action is given in terms of a single holomorphic prepotential. As in the N=1 superpotential case, this prepotential is exactly determined using the symmetries, holomorphicity and duality properties of the model in question. Seiberg and Witten [4] secured the form of the prepotential (and hence the Kähler metric on the quantum moduli space) in a N=2, SU(2) strongly interacting gauge theory which is spontaneously broken to a low energy N=2, U(1) gauge theory. In terms of N=1 superfields, this low energy action has the form

$$
\begin{aligned}
\Gamma_{N=2}[\phi,\bar{\phi},V] &= \frac{1}{8\pi i}\left(\int dV\left[\mathcal{F}_{\varphi}(\varphi)\bar{\varphi}-\bar{\mathcal{F}}_{\bar{\varphi}}(\bar{\varphi})\varphi\right]\right. \\
&\left. +\frac{1}{2}\int dS\mathcal{F}_{\varphi\varphi}(\varphi)W^{\alpha}W_{\alpha}-\frac{1}{2}\int d\bar{S}\bar{\mathcal{F}}_{\bar{\varphi}\bar{\varphi}}(\bar{\varphi})\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\right), \qquad (1)
\end{aligned}
$$

where $(\bar{\varphi})$ φ is a neutral (anti-) chiral superfield, the N=2 partner of the abelian gauge field V. Here the subscripts denote differentiation with respect to that field so that, for example, $\mathcal{F}_{\varphi}=\frac{\partial\mathcal{F}}{\partial\varphi}$. The chiral field strength is defined by $W_{\alpha}=-\frac{1}{4}\bar{D}\bar{D}(e^{-V}D_{\alpha}e^{V})=-\frac{1}{4}\bar{D}\bar{D}D_{\alpha}V$, with a similar definition, $\bar{W}_{\dot{\alpha}}=-\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V$, for the anti-chiral field strength. The holomorphic prepotential $\mathcal{F}(\varphi)$ then determines the Kähler potential $K(\varphi,\bar{\varphi})=\frac{1}{4\pi}\mathrm{Im}[\mathcal{F}_{\varphi}(\varphi)\bar{\varphi}]$ and

hence the Kähler metric

$$g(\varphi,\bar{\varphi}) = K_{\varphi\bar{\varphi}}(\varphi,\bar{\varphi}) = \frac{1}{4\pi}\mathrm{Im}\ \mathcal{F}_{\varphi\varphi}. \tag{2}$$

Seiberg and Witten constructed $g(\varphi,\bar{\varphi})$ for a strongly interacting N=2 SU(2) Yang-Mills theory. Analogous results have also been obtained in other models which include matter fields as well as different gauge groups [5].

A general low energy effective action for an arbitrary N=1 supersymmetric theory containing these same fields can be written as

$$\begin{aligned} \Gamma[\phi,\bar{\phi},V] &= \int dV K(\varphi,\bar{\varphi}) + \frac{1}{2}\int dS f(\varphi) W^\alpha W_\alpha + \frac{1}{2}\int d\bar{S}\bar{f}(\bar{\varphi})\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} \\ &\quad +\kappa\int dV V + \int dS P(\varphi) + \int d\bar{S}\bar{P}(\bar{\varphi}). \end{aligned} \tag{3}$$

Here the Kähler potential, $K(\varphi,\bar{\varphi})$, the effective gauge couplings, $f(\varphi)$ and $\bar{f}(\bar{\varphi})$, and the superpotentials, $P(\varphi)$ and $\bar{P}(\bar{\varphi})$, are arbitrary, in general, unrelated functions of their respective arguments while the linear in V Fayet-Iliopolous term has an arbitrary, but field independent, coupling constant κ. These functions and couplings evolve under renormalization group transformations operating at the scale at which the effective theory is defined. As is well known, the superpotential does not evolve independently but only according to the anomalous dimension of the chiral fields. In this topologically trivial model, if it initially vanishes, the superpotential remains zero. Likewise, the Fayet-Iliopolous term has no independent additive radiative corrections. Consequently its zero value is stable as the system evolves. Furthermore, after choosing these terms to vanish, we shall use the low energy truncation of the Wilson renormalization group equations to establish that the general N=1 supersymmetric effective action (3) evolves toward satisfying the N=2 relation between the Kähler potential and the effective gauge

couplings given by $g(\varphi, \bar{\varphi}) = f(\varphi) + \bar{f}(\bar{\varphi})$ as the theory flows toward the infrared stable trivial fixed point. Such a renormalization group behavior in N=1 SUSY theories is reminiscent of previous studies[6] of various different non-abelian gauge models which exhibit a manifold of infrared attractive stable N=2 supersymmetric non-trivial fixed points towards which the theory evolves. The present abelian N=1 model exhibits a similar behavior except in this case, the N=2 fixed point is the trivial one.

The Wilson renormalization group equation (WRGE) describes the change of the (Wilson) effective action as the quantum mechanical effects of the degrees of freedom with momentum differentially below the running cutoff $\Lambda(t) = e^{-t}\Lambda$ are taken into account [7][8]. It is secured by demanding that correlation functions remain unchanged as the degrees of freedom in the differential shell are integrated out. As such, the action satisfying the WRGE can be used to describe the physics on all scales below $\Lambda(t)$. Making a momentum expansion of the general effective action and retaining all terms containing two or less space-time derivatives, then the WRGE describes the flow of the low energy action (3). Although such an action is a trunction of the space of all possible actions, its self-radiative corrections will be consistently determined by this method, independent of the strength of the interaction. Integrating over vector and chiral superfields with momentum between $e^{-t}\Lambda(t)$ and $\Lambda(t)$ while re-scaling all dimensionful parameters by $\Lambda(t)$ and all fields according to their anomalous dimensionality, we secure the Wilson renormalization group equations for the action (3). These radiative corrections arise only from the one loop one particle irreducible diagrams with all internal lines having momentum at the cutoff [9].

The superpotential receives no radiative corrections and its Wilson renor-

malization group equation is simply given by

$$\begin{aligned}\frac{\partial P}{\partial t} &= 3P - (1-\gamma)\varphi P_{\varphi} \\ \frac{\partial \bar{P}}{\partial t} &= 3\bar{P} - (1-\gamma)\bar{\varphi}\bar{P}_{\bar{\varphi}}.\end{aligned} \tag{4}$$

Here γ is the anomalous dimension for the (anti-) chiral superfield. It follows that if the superpotential vanishes at $t = 0$, it will remain zero for all $t > 0$. This is simply the Wilson renormalization group equation verification of the non-renormalization theorem for the superpotential in this model. Likewise in this model, the Fayet-Iliopoulous term receives no radiative corrections. Thus if initially zero, it also remains zero.

On the other hand, the Kähler potential does, in general, receive radiative corrections. These can be computed by evaluating the graphs in which both chiral fields and the abelian gauge fields propagate in the loop with external legs carrying zero momentum. Once again defining the Kähler metric as $g(\varphi, \bar{\varphi}) = K_{\varphi\bar{\varphi}}(\varphi, \bar{\varphi})$, we find the Wilson renormalization group equation for g takes the form

$$\begin{aligned}\frac{\partial g}{\partial t} &= 2\gamma g - (1-\gamma)\varphi g_{\varphi} - (1-\gamma)\bar{\varphi} g_{\bar{\varphi}} \\ &\quad + \frac{1}{8\pi^2}\left(\frac{g_{\varphi} g_{\bar{\varphi}}}{g^2} - \frac{g_{\varphi\bar{\varphi}}}{g} - \frac{f_{\varphi}\bar{f}_{\bar{\varphi}}}{(f+\bar{f})^2}\right).\end{aligned} \tag{5}$$

In obtaining the above result, we have employed an R_ξ gauge so that the action is augmented by the term

$$\Gamma_\xi[V] = \frac{\xi}{8}\int dV(DDV)(\bar{D}\bar{D}V). \tag{6}$$

Note that the gauge fixing parameter ξ receives no radiative corrections in this simple abelian model. As such, it does not contribute to the Kähler

potential radiative corrections. Since the fields have been rescaled at each t during the renormalization flow according to their full scaling dimensions, the chiral field anomalous dimension can be extracted by evaluating equation (5) at $\varphi = \bar{\varphi} = 0$ where $g|_{\varphi=\bar{\varphi}=0} = 1$ and $f|_{\varphi=0} = \bar{f}_{\bar{\varphi}=0} = \frac{1}{2}$. So doing, we find

$$2\gamma = \frac{1}{8\pi^2}\left[g_{\varphi\bar{\varphi}} - g_{\varphi}g_{\bar{\varphi}} + f_{\varphi}\bar{f}_{\bar{\varphi}}\right]|_{\varphi=\bar{\varphi}=0}. \tag{7}$$

Using Eq. (5), the Kähler potential is seen to satisfy the Wilson renormalization group equation

$$\frac{\partial K}{\partial t} = 2K - (1-\gamma)\varphi K_{\varphi} - (1-\gamma)\bar{\varphi}K_{\bar{\varphi}} - \frac{1}{8\pi^2}\ln(\frac{K_{\varphi\bar{\varphi}}}{f+\bar{f}}). \tag{8}$$

Note that the Kähler potential is only determined up to additive holomorphic chiral and anti-chiral functions so that K and $K + F(\varphi) + \bar{F}(\bar{\varphi})$ give rise to the same metric and hence produce the same physics [10][11] .

Next turning to the running of the holomorphic effective gauge couplings f and $\bar{f}$, here one again finds that the radiative loop corrections identically vanish so that they also evolve only according to the anomalous dimensionality of the fields. The resultant Wilson renormalization group equations take the simple form

$$\begin{aligned} \frac{\partial f}{\partial t} &= 2\gamma_V f - (1-\gamma)\varphi f_{\varphi} \\ \frac{\partial \bar{f}}{\partial t} &= 2\gamma_V \bar{f} - (1-\gamma)\bar{\varphi}\bar{f}_{\bar{\varphi}}, \end{aligned} \tag{9}$$

where the photon anomalous dimension is denoted as γ_V. By again evaluating these equations at zero fields, we immediately glean that $\gamma_V = 0$. Hence the scaling of effective gauge couplings is secured as

$$\begin{aligned} f(\varphi, t) &= f(e^{\int_0^t dt(\gamma-1)}\varphi, 0) = \tau(e^{\int_0^t dt(\gamma-1)}\varphi) \\ \bar{f}(\bar{\varphi}, t) &= \bar{f}(e^{\int_0^t dt(\gamma-1)}\bar{\varphi}, 0) = \bar{\tau}(e^{\int_0^t dt(\gamma-1)}\bar{\varphi}), \end{aligned} \tag{10}$$

where $\tau(\varphi)$ and $\bar{\tau}(\bar{\varphi})$ are the initial holomorphic effective gauge couplings.

Now consider Eq. (5) satisfied by the Kähler metric. Note that when

$$g(\varphi, \bar{\varphi}) = f(\varphi) + \bar{f}(\bar{\varphi}), \tag{11}$$

so that $g_\varphi = f_\varphi$ and $f_{\bar{\varphi}} = \bar{f}_{\bar{\varphi}}$ while $g_{\varphi\bar{\varphi}} = 0$, then the loop corrections identically vanish. As such this can be viewed as a fixed relation of the Wilson renormalization group equation. Moreover, when this functional relation is satisfied, it also follows that $\gamma = 0$. We thus secure the metric form

$$g(\varphi, \bar{\varphi}) = f(\varphi) + \bar{f}(\bar{\varphi}) = \tau(e^{-t}\varphi) + \bar{\tau}(e^{-t}\bar{\varphi}). \tag{12}$$

Note that an immediate consequence of this result is that if the action is initially N=2 supersymmetric, then it remains N=2 supersymmetric with a holomorphic prepotential $\mathcal{F}$ whose functional form is fixed and which evolves into the infrared according to the above naive dimensional scaling of the fields. For this N=2 abelian model, the form of the next higher order term (p^4) in the momentum expansion of the action has also been investigated [12]. Its structure requires the introduction of an additional real analytic N=2 supersymmetric function of the fields. On the other hand, an explicit superspace one-loop perturbative calculation [13][14][15] for an N=2 non-abelian gauge theory which also contains a superpotential exhibits a breakdown of the manifest N=2 invariance and the Kähler potential cannot be written in terms of a single holomorphic function.

To establish that the N=2 supersymmetric relation $g = (f + \bar{f})$ is an attractive infrared stable fixed condition, the Wilson equations (5) and (9) are expanded for small field values. So doing, it follows that the ratios

$$r \equiv \frac{g_\varphi|_{\varphi=\bar{\varphi}=0}}{f_\varphi|_{\varphi=0}}$$

$$\bar{r} \equiv \frac{\bar{g}_{\bar{\varphi}}|_{\varphi=\bar{\varphi}=0}}{\bar{f}_{\bar{\varphi}}|_{\bar{\varphi}=0}}, \tag{13}$$

satisfy the renormalization group equations

$$\begin{aligned} \frac{1}{r}\frac{dr}{dt} &= -\frac{1}{8\pi^2}[f_\varphi \bar{f}_{\bar{\varphi}}]|_{\varphi=\bar{\varphi}=0}\left[(r\bar{r}-1)+2(r\bar{r}-\frac{1}{r})\right] \\ \frac{1}{\bar{r}}\frac{d\bar{r}}{dt} &= -\frac{1}{8\pi^2}[f_\varphi \bar{f}_{\bar{\varphi}}]|_{\varphi=\bar{\varphi}=0}\left[(r\bar{r}-1)+2(r\bar{r}-\frac{1}{\bar{r}})\right]. \end{aligned} \tag{14}$$

The N=2 relation $g = f + \bar{f}$ requires that $g_\varphi|_{\varphi=\bar{\varphi}=0} = f_\varphi|_{\varphi=0}$ and $g_{\bar{\varphi}}|_{\varphi=\bar{\varphi}=0} = \bar{f}_{\bar{\varphi}}|_{\bar{\varphi}=0}$. These in turn correspond to the values $r = \bar{r} = 1$ which are clearly fixed points of Eq. (14). Considering small deviations from the fixed point values, so that $r = 1 + \epsilon$ and $\bar{r} = 1 + \bar{\epsilon}$, we find that

$$\begin{aligned} \frac{dr}{dt} &= -\frac{1}{8\pi^2}[f_\varphi \bar{f}_{\bar{\varphi}}]|_{\varphi=\bar{\varphi}=0}\,[5\epsilon + 3\bar{\epsilon}] \\ \frac{d\bar{r}}{dt} &= -\frac{1}{8\pi^2}[f_\varphi \bar{f}_{\bar{\varphi}}]|_{\varphi=\bar{\varphi}=0}\,[5\bar{\epsilon} + 3\epsilon]\,. \end{aligned} \tag{15}$$

Hence, the N=2 fixed point $r = \bar{r} = 1$ is indeed attractive. The N=1 supersymmetric theory evolves towards the N=2 supersymmetric theory characterized by the relation $g = (f + \bar{f})$.

In summary, we have examined the structure of the low energy Wilson effective action containing abelian vector and neutral chiral superfields as the theory evolves into the infrared. The self-radiative corrections of this N=1 supersymmetric theory drive the low energy degrees of freedom to a N=2 symmetric theory. If the action is initially N=2 supersymmetric, then it remains N=2 supersymmetric with a holomorphic prepotential $\mathcal{F}$ whose functional form is fixed and which evolves into the infrared according to the naive dimensional scaling of the fields.

This work was supported in part by the U.S. Department of Energy under grant DE-FG02-91ER40681 (Task B).

References

[1] K. Fujikawa and W. Lang, *Nucl. Phys.* **B88** (1975) 61.

[2] M.T. Grisaru, M. Roček and W. Siegel, *Nucl. Phys.* **B159** (1979) 429.

[3] N. Seiberg, *Phys. Lett.* **B318** (1993) 469; **B206** (1988) 75.

[4] N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994) 19; **B431** (1994) 484.

[5] A. Klemm, W. Lerche, S. Yankielowicz and S. Theisen, *Phys. Lett.* **B344** (1995) 169; P.C. Argyres and A.E. Faraggi, *Phys. Rev. Lett.* **74** (1995) 3931; U.H. Danielsson and B. sundborg, *Phys. Lett.* **B358** (1995) 273; A. Brandhuber and K. Landsteiner, *Phys. Lett.* **B358** (1995) 73; P.C. Argyres, M.R. Plesser, N. Seiberg and E. Witten, *Nucl. Phys.* **B461** (1996) 71; P.C. Argyres and A.D. Shapere, *Nucl. Phys.* **B461** (1996) 437.

[6] R.G. Leigh and M.J. Strassler, *Nucl. Phys.* **B447** (1995) 95; M.J. Strassler, *Manifolds of Fixed Points and Duality in Supersymmetric Gauge Theories*, hep-th/9602021.

[7] K.G. Wilson, *Phys. Rev.* **B4** (1974) 3174; K.G. Wilson and J.B. Kogut, *Phys. Rep.* **12** (1975) 75.

[8] F.J. Wegner and A. Houghton, *Phys. Rev.* **A8** (1973) 401; F.J. Wegner, in **Phase Transitions and Critical Phenomena**, vol. 6, ed. C. Domb and M.S. Green (Academic Press, New York, 1976) 8; in **Trends in Elementary Particle Theory**, vol. 37 (Springer, Berlin, 1975) 171.

[9] T.E. Clark, B. Haeri and S.T. Love, *Nucl. Phys.* **B402** (1993) 628.

[10] J. Bagger and E. Witten, *Phys. Lett.* **B118** (1982) 103.

[11] T.E. Clark and S.T. Love, *Nucl. Phys.* **B232** (1984) 306; *Nucl. Phys.* **B254** (1985) 569.

[12] M. Henningson, *Nucl. Phys.* **B458** (1996) 445.

[13] M.T. Grisaru, M. Roček and R. von Unge, *Effective Kähler Potentials*, hep-th/9605149.

[14] B. de Wit, M.T. Grisaru and M. Roček, *Phys. Lett.* **B374** (1996) 297.

[15] A. Pickering and P. West, *The One Loop Effective Super-Potential and Non-Holomorphicity*, hep-th/9604147.